# M.Sc. Sem II, Paper V
## Graph Theory (Continued)

(II) By allowing E to contain both directed or undirected edges, we obtain a Mixed Graph.

(III) By allowing repeated elements in the set of edges. i.e. by replacing E with a multiset, we obtain a Multi Graph.

(IV) By allowing edges to connect a vertex to itself (a loop), we obtain Pseudo Graphs

(V) By allowing the edges to be arbitrary subsets of vertices, not necessarily of size two, we obtain Hypergraphs.

(VI) By allowing V and E to be infinite sets, we obtain infinite graphs.

Simple graph - A graph that is a finite undirected graph without loops and multiple edges.

* The sets of vertices and edges of a graph G will be denoted V(G) and E(G), respectively.

* for notational convenience, instead of representing an edge by {a, b} we shall denote it by ab.

Adjacent, Neighbour, vertex Degree

Let u, v be two vertices of a graph G.

(i) If $uv \in E(G)$, then u, v are said to be adjacent, in which case we also say that u is connected to v or u is a neighbour of v. If $uv \notin E(G)$, then

u and v are nonadjacent (not connected non-neighbours),

② The neighbourhood of a vertex $v \in V(G)$, denoted $N(v)$, is the set of vertices adjacent to v, i.e. $N(v) = \{u \in V(G) \mid vu \in E(G)\}$. The closed neighbourhood of v is denoted and defined as $N[v] = N(v) \cup \{v\}$.

③ If $e = uv$ is an edge of G, then e is incident to u and v. We also say that u and v are the endpoints of e.

④ The degree of $v \in V(G)$, denoted $\deg(v)$, is the number of edges incident to v. Alternatively $\deg(v) = |N(v)|$. If $\deg(v) = 0$, then vertex v is called isolated. If $\deg(v) = 1$, then vertex v and the only edge incident to v are called pendant. The maxm. vertex degree and the minm. vertex degree in a graph G are denoted by $\Delta(G)$ and $\delta(G)$, respectively.

PATH of Graph - A path in a graph is a sequence of distinct vertices $v_1, v_2, \ldots, v_k$ such that $v_i v_{i+1}$ is an edge for each $i = 1, \ldots, k-1$. The length of a path P is the number of edges in P. A chord in a path is an edge connecting two non-consecutive vertices. A chordless path is a path without chords.

A graph G is connected if every pair of distinct vertices is joined by a path. otherwise it is disconnected. The distance between two vertices a and b, denoted dist(a,b), is the length of a shortest path joining them.

The diameter of a connected graph, denoted diam(G), is $\max_{a,b \in V(G)} dist(a,b)$.